

$\Phi \in H_1 \otimes H_2$, $\dim H_i = 2$.

Assertion: A n.d.s. cond. for

$\nexists P_2 \exists P_1 \ni \langle P_1 P_2 \rangle_{\Phi} = \langle P_1 \rangle_{\Phi} \neq 0$
is that Φ is entangled.

Suppose Φ entangled: $\Phi = d_1 u_1 \otimes v_1 + d_2 u_2 \otimes v_2$

$\{u_i\}$ o.n., $\{v_i\}$ o.n., $|d_1|^2 + |d_2|^2 = 1$, $d_i \neq 0$.

Given P_2 there is a basis $w_1, w_2 \ni$

$$P_2 w_1 = w_1, \quad P_2 w_2 = 0.$$

$$v_1 = c_1 w_1 + c_2 w_2$$

$$v_2 = d_1 w_1 + d_2 w_2$$

c_1, d_1 cannot both be 0,
for then orthogonality of
 v_1, v_2 fails.

$$\begin{aligned} \Phi &= (d_1 c_1 u_1 + d_2 d_1 u_2) \otimes w_1 \\ &\quad + (d_1 c_2 u_1 + d_2 d_2 u_2) \otimes w_2. \end{aligned}$$

$$d_1 c_2 u_1 + d_2 d_2 u_2 \neq 0.$$

Let P_1 be the projector on H_1 which has this vector as eigenvector with eigenvalue 0. The vectors orthogonal to it have eigenvalue 1; let z_1 be such.

$$\begin{aligned} \langle P_1 P_2 \rangle_{\Phi} &= |(z_1, d_1 c_1 u_1 + d_2 d_1 u_2)|^2 \\ &= \langle P_1 \rangle_{\Phi} \neq 0. \end{aligned}$$

If Φ is not entangled, d_1 or d_2 is 0.
Say d_1 . Then there can be a P_2 such that $d_1 c_2 u_1 + d_2 d_2 u_2 = 0$. Then for any P_1 , $\langle P_1 P_2 \rangle_{\Phi} = 0$.